which is the proportionality factor in the relation $\arg K(ka) = \alpha\omega$. The shapes of the curves are identical for the investigated bandwidths $2\Delta f$; an increase in the bandwidth corresponds to an increase in the slope of the sides of the envelope, i.e., greater approximation to a rectangular shape. The calculations show that disparities between the shapes of $\overline{Y}(\tau)$ and $\overline{X}(\tau)$ set in when $2\Delta f$ is increased, and the larger the value of $2\Delta f$, the greater will be the disparities. They are obviously caused by breakdown of the condition $|K(ka, t_0)| \simeq$ const, $\omega_H \leq \omega \leq \omega_b$.

We note, finally, that the most commonly used sonar characteristic of an object is the target strength, which is defined by the relation $TS = 10 \log \sigma/4\pi|_{r=1\ m}$, dB; in the case of a reference sphere of optimum radius a_{opt} it is given on the basis of Eq. (6) by the expression

$$TS \simeq 20 \lg |K((ka)_{opt}, t_0)|, \text{ dB}. \tag{9}$$

Comparative calculations of the target strengths using Eq. (2) and the approximate expression (9) for various values of $(ka)_{opt}$, which are controlled by f_0 and $c(t)$, indicate that they agree to within hundredths of a decibel. A comparison of these results with the data of Ref. 4, in which the target strength of electrolytic copper spheres is calculated for various Simrad EK400/38 sonar systems, shows that they are practically identical: According to the data of Ref. 4, $TS = -33.6 \pm 0.1$ dB at $f_0 = 38$ kHz; according to the calculations in the present study, $TS = -33.4$ dB.

[1] R. J. Urick, Principles of Underwater Sound, 3rd Ed., McGraw-Hill, New York (1983).

[2] L. R. Dragonette, J. Acoust. Soc. Am. 69, 1186 (1981).

[3] K. I. Yudanov, I. L. Kalikhman, and V. D. Tesler, Manual of Underwater Acoustic Recording [in Russian], VNIRO MRKh SSSR, Moscow (1984).

[4] K. G. Foote, J. Acoust. Soc. Am. 71, 742 (1982).

[5] R. Hickling, J. Acoust. Soc. Am. 34, 1582 (1962).

[6] W. G. Neubauer, R. H. Vogt, and L. R. Dragonette, J. Acoust. Soc. Am. 55, 1123 (1974).

[7] K. G. Foote, J. Acoust. Soc. Am. 73, 1054 (1983).

[8] A. L. Zinov'ev, and L. I. Fillipov, Introduction to the Theory of Signals and Networks [in Russian], Vysshaya Shkola, Moscow (1968).

[9] G. E. Forsythe, M. A. Malcolm, and C. B. Moler, Computer Methods for Mathematical Computations, Prentice-Hall, Englewood Cliffs, NJ (1977).

Translated by J. S. Wood

Modeling of mode interaction in a two-dimensionally inhomogeneous waveguide

S. V. Burenkov and O. E. Popov

N. N. Andreev Acoustics Institute, Academy of Sciences of the USSR

(Submitted August 24, 1988)

Akust. Zh. **35**, 590–595 (July–August 1989)

The modeling of mode interaction in an irregular waveguide by the parabolic equation method is discussed. The results are compared with theoretical calculations for a simple model of the medium. The results of mode interaction at the exit from the irregular section of the waveguide are plotted as a function of the phase difference between the excitation coefficents at the entrance to the irregular section.

The main tool used to solve problems of sound propagation in two-dimensionally inhomogeneous waveguides is the parabolic equation (PE) method, it lags behind the classical analytical methods of normal modes and rays in visualization of the final results and simplicity of their interpretation.

The objective of the present article is to find a way to consolidate the universality of numerical methods with the familiarity and clarity of analytical methods. This objective is achieved by calculating vertical profiles of the sound field in planar two-dimensionally inhomogeneous waveguides according to a program that implements the PE method and then expanding the profiles in mode eigenfunctions of comparison waveguides.

This computational scheme has the following useful properties in contrast with direct application of the PE method to calculate the sound field: First, a large number of important practical problems have been solved in the mode approximation for plane-layered media; the proposed procedure permits these results to be extended to the case of an irregular waveguide (albeit in numerical form). Second, several approximate methods have been developed for the investigation of mode interaction in ocean waveguides; the proposed procedure can be used to determine how and under what conditions mode interaction takes place when the adiabatic approximation is valid, etc.

Avilov's program,[1] which implements a refined wide-angle parabolic approximation, is used to calculate the sound field. The eigenfunctions of the comparison waveguides are calculated according to the well-known program of Vagin and Mal'tsev.[2] The field is expanded in complex-valued orthonormal eigenfunctions $\psi_n(z)$ with allowance for waveguide

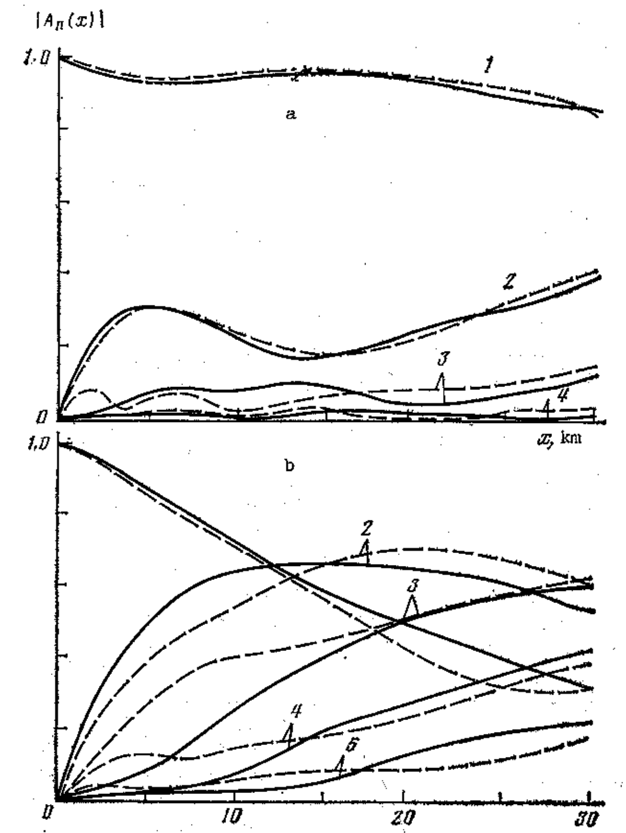

FIG. 1. Moduli of the excitation coefficients $|A_n(x)|$ vs distance in a wedge with the first mode excited at the entrance to the wedge. a) f = 25 Hz; b) f = 65 Hz. The mode orders are indicated alongside the curves. The solid curves are calculated according to the parabolic equation method, and the dashed curves are calculated analytically.

sections in which these functions decay exponentially, thus ensuring a high degree of orthogonality in the numerical calculations. The calculated values for the examples discussed below are

$$|\langle\psi_n|\psi_m\rangle| = \left|\int \psi_n(z)\psi_m(z)\,dz\right| = \begin{cases} <4\cdot 10^{-6} & \text{for} \quad n\neq m \\ \approx 1.0 & \text{for} \quad n=m \end{cases},$$

where m and n are mode orders.

In order for one or more modes to be excited in the waveguide, the field in the initial cross section is specified with a vertical distribution corresponding to one mode or a sum of several modes $p(x_0, z) = \sum_n A_n \exp(i\varphi_n)\psi_n(z)$, where A_n and ϕ_n are the excitation coefficient and phase angle of the n-th mode respectively, and x_0 is the coordinate of the initial cross section.

To test the performance of the algorithm, the first mode with excitation coefficient equal to unity was excited at the entrance to a three-mode waveguide of constant depth, and then one horizontal profile and several vertical profiles of the field were calculated. Upon expansion of the vertical profiles of the field in eigenfunctions over an entire range, equal to 200 thicknesses of the waveguide, it was found that the excitation coefficient of the first mode remained almost equal to unity, while the coefficients for the second and third modes did not exceed 0.1. This result indicates that only the first waveguide mode is generated efficiently, and it propagates without any variation of its waveform; the energy of the other modes does not exceed one percent of the total energy of the field in this case. The Fourier transform with respect to the horizontal profile also yields one maximum in the wave number spectrum, which corresponds to the phase velocity of the generated first mode. The amplitudes of the other modes do not exceed the side lobes of the principal maximum in this case. Thus, two independent tests demonstrate the correctness of the numerical implementation of the proposed procedure.

We use the indicated algorithm to investigate mode interaction in downslope sound propagation along a wedge. The problem of mode interaction over a sloping bottom is treated extensively in the literature (see, e.g., Ref. 3), including modeling by the PE method[4,5]; on the whole, however, the problem has not been adequately investigated.

The model of the medium represents an isovelocity (c_1 = 1450 m/s) wedge situated on a fluid half-space with c_2 = 1650 m/s and slope angle α = 2°; the wedge is bounded on the left and right by waveguides of constant depth equal to 350 and 1400 m, respectively. A transition layer of thickness 30 m with a linearly varying $c^{-2}(z)$ is situated between the water and the bottom. The density of the wedge and the underlying space is 1000 kg/m³. The cross section in which the selected set of modes is excited is located in the left waveguide at a distance L from the start of the wedge (x = 0); L is varied from 0 to 40 km.

The solid curves in Figs. 1 and 2 represent the moduli of the excitation coefficients $|A_n(x)|$ of modes propagating downslope when only the first mode or only the second mode with unit amplitude is excited at the entrance to the wedge (L = 0) at frequencies of 25 and 65 Hz, respectively. We see that both the first mode and the second mode propagate with scarcely any interaction at a frequency of 25 Hz (Figs. 1a and 2a). The pattern differs sharply at a frequency of 65 Hz (Figs. 1b and 2b); mode interaction is very strong, and the initially excited mode contains less energy than the adjacent modes at the exit from the wedge, i.e., at x = 30 km.

These results can be explained, e.g., by analytical modeling of the mode-interaction process.[6] In this method the field at the entrance to the section with the sloping bottom is expanded in modes of the exact solution with phase surfaces in the form of arcs of a circle centered at the vertex of the wedge formed by the continuation of the sloping bottom to the point of intersection with the surface. These modes propagate over the sloping bottom without interacting, and the field generated by them at the "exit" from the section with the sloping bottom decomposes into ordinary plane modes of a constant-depth waveguide. We find in this case that

$$k\alpha h(x) < 2, \tag{1}$$

where k is the wave number, h(x) is the depth, and α is the slope angle of the bottom in radians; each entrant mode efficiently generates only one mode of the exact solution, so that mode interaction is practically nonexistent during propagation, i.e., only half the original energy is left in the entrant mode.

Proceding from this fact, we chose the param-

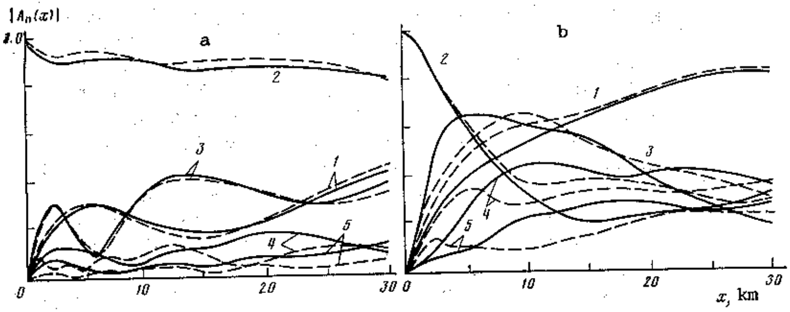

FIG. 2. Moduli of the excitation coefficients $|A_n(x)|$ vs distance in a wedge with excitation of the second mode at the entrance to the wedge. a) f = 25 Hz; b) f = 65 Hz. The solid curves are calculated according to the PE method, and the dashed curves are calculated analytically. The mode orders are indicated alongside the curves.

eters in the numerical experiments (Figs. 1 and 2) in such a way that condition (1) was satisfied up to x = 15 km at a frequency of 25 Hz (Figs. 1a and 2a) and was satisfied only at the point x = 0 km at 65 Hz. Although the mode interaction pattern is complicated by interference beats between individual modes, it is evident in the simplest case of excitation of the first mode at the entrance (Fig. 1a) that a "nonrecoverable" energy loss of the entrant mode sets in at about 15 km, along with a corresponding smooth constant growth of the second-mode amplitude. When the second mode is excited (Fig. 2a), energy exchange takes place simultaneously with the first and with the third modes. Consequently, interference effects are more pronounced, and irreversible dissipation of the entrant mode energy begins at about 20 km.

Conditions of the type (1) have been obtained repeatedly by different means in different papers,[6,8] and they differ only in the value of the constant on the right-hand side. The results of our calculations of $|A_n(x)|$ by the PE method show that the crossover between "almost adiabatic" and "explicitly nonadiabatic" propagation in the above-indicated sense may be a highly conditional boundary but it does in fact occur in the vicinity of $kh(x)\alpha \sim 2$.

The simplicity of the adopted model enables us to determine the functions $A_n(x)$ for it by the analytical modeling method proposed in Ref. 6. The above model of a wedge with a permeable bottom is now replaced by a wedge with perfectly compliant boundaries and a depth $h'(x) = h(x) + \Delta h$, where $\Delta h = c_1 c_2 / 4f(c_2^2 - c_1^2)^{0.5}$, making it possible to satisfy the boundary conditions for h(x) to within $O(\alpha_2)$ (Ref. 9). The modeling results are represented by the dashed curves in Figs. 1 and 2. We see that the curves obtained by the different methods are in good agreement, particularly for the energy-carrying modes. The agreement of the results obtained by these different methods attests in favor of both the analytical modeling method[6] and the refined wide-angle PE program used here.[1]

Downslope sound propagation along a wedge can be interpreted qualitatively on the basis of the fact that a mode in a homogeneous waveguide can be represented by a superposition of two plane waves or by systems of parallel rays.

Thus, let us suppose that the n-th mode is generated at the entrance to the wedge (x = 0), corresponding to two systems of parallel rays propagating at the Brillouin angles $\pm B_n(0)$ relative to the horizontal. If $|B_n(0)| < \alpha$, such rays will not interact with the bottom and will not change the grazing angle. As the depth h(x) increases with x, the Brillouin angle $B_n \approx \pi n/kh(x)$ corresponding to the n-th mode in the cross section x decreases, and the time comes when the system of rays propagating at the angle $B_n(0)$ will more nearly correspond to the (n + 1)-st mode, i.e., $B_n(0) \approx B_{n+1}(x)$.

Consequently, the same system of rays with a constant grazing angle will be interpreted in different cross sections of the wedge as sifferent mode orders, depending on the depth. This situation can be regarded conditionally as the mechanism of energy transfer from lower to higher modes. It is evident in Fig. 1b that the excitation coefficients of the second and third modes at 65 Hz in the case of excitation of the first mode at the entrance to the wedge are close to attaining their maxima at the abscissas x = 11 km and x = 21 km, where the line $y = B_0(0)$ intersects the curves $y = B_2(x)$ and $y = B_3(x)$ (Fig. 3), i.e., the first-mode energy is converted into energy of the second and third modes, respectively.

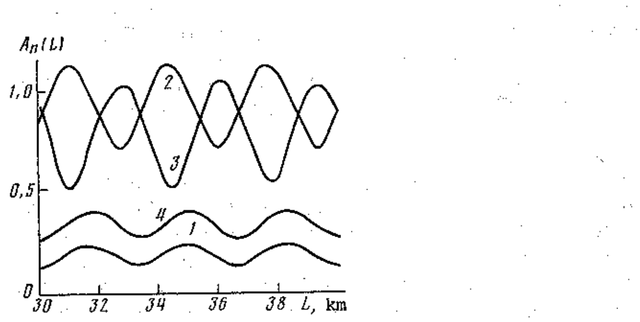

FIG. 3. Angles $B_n(x)$ vs distance in a wedge. The mode orders are indicated alongside the curves.

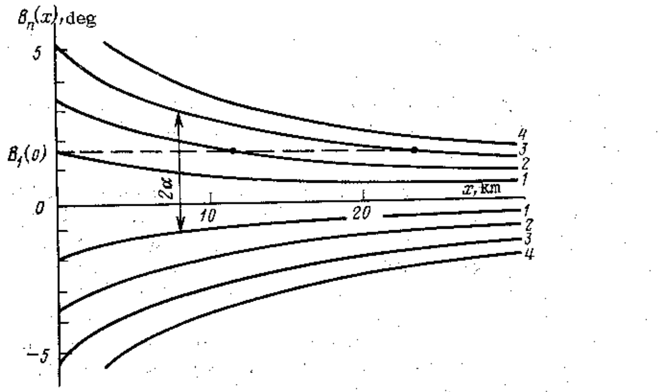

FIG. 4. Moduli of the excitation coefficients $|A_n(x)|$ at the exit from a wedge vs length of the planar section L. The mode orders are indicated alongside the curves.

If the angle $B_n(0) > \alpha$, the grazing angle of the system of rays decreases by 2 upon hitting the bottom. Rays with this grazing angle $[B_n(0) - 2\alpha]$ correspond to the lowest mode, i.e., energy is transferred from higher to lower modes. The excitation coefficient of all modes except the first and third (dashed curves in Fig. 2b) vary only slightly at a distance $x \geq 8$ km; we can therefore assume that energy exchange takes place essentially between the first and third modes, the third giving up energy and the first acquiring energy. The condition $B_3(x) - 2\alpha \approx -B_1(x)$ is observed in the neighborhood of $x = 8$ km (see Fig. 3).

Another important phenomenon that has not been recognized previously in the investigation of the given problem is the fact that when a group of modes is excited at the entrance to the wedge, the result of mode interaction at the exit from the wedge depends strongly on the phases of the modes at the entrance.

In fact, because of the linearity of the acoustical equations, it is readily shown that if the pressure distribution $p_1(z)$ at the entrance to the wedge gives the excitation coefficient ${}_1A_n$ of the n-th mode at the exit and if the distribution $p_n(z)$ gives the excitation coefficient ${}_2A_n$, then the distribution $p(z) = p_1(z) + p_2(z)e^{i\phi}$ gives the excitation coefficient $A_n = {}_1A_n + {}_2A_n e^{i\phi}$, and the modulus $|A_n|$ varies from $|{}_1A_n + {}_2A_n|$ to $|{}_1A_n - {}_2A_n|$, depending on the phase difference.

In practice, this result implies that when only the n-th mode is excited at the entrance to the wedge under condition (1), adjacent modes will provide a small (energywise) contribution to the field at the exit ($|A_{k\pm 1}|^2 \ll |A_k|^2$). On the other hand, if two adjacent mode orders k and k + 1 with comparable amplitudes and a phase difference ϕ are excited, the additional contributions from the k-th to the (k + 1)-st mode and vice versa are summed with the phase ϕ at the exit from the same wedge, inducing a major redistribution of energy between the k-th and (k + 1)-st modes, depending on ϕ.

We perform the following numerical experiment to illustrate this assertion: We excite the second and third modes with amplitudes 1.0 and 0.9, respectively, at a frequency of 25 Hz in the left waveguide at a distance L from the start of the wedge ($x = 0$). Varying L from 30 to 40 km, we obtain a variation of ϕ by 6π ($\pi(L) = (\xi_2 - \xi_3)L \approx 5\pi L c_1/(4h^2(0)f)$, where f is the frequency, ξ_n is the wave number of the n-th mode, and $h(0) = 350$ m. Figure 4 shows the excitation coefficients of the first four modes as a function of L in the cross section $x = 15$ km [condition (1) holds up to this cross section]. The dependence of the excitation coefficients on L has a periodic behavior, and the variation of $\phi(L)$ by 6π corresponds to three periods in Fig. 4; despite the satisfaction of condition (1), the energy of the third mode varies fourfold.

We infer from the foregoing discussion that the condition (1) ensures the smallness of mode interaction (in energy terms) over a sloping bottom only when a single mode is excited; the excitation of a group of modes requires a stricter condition, which limits the range of applicability of the adiabatic approximation in the given problem to the lowest frequencies. The specific form of this condition will depend on the accuracy requirements imposed on the field calculated in the adiabatic approximation.

Summarizing the modeling study, the proposed method for the investigation of mode interaction by the PE method holds up under comparison with the exact solution, opening broad horizons for its application in various problem. Its application to a waveguide with a sloping bottom made it possible to determine the principal laws of mode interaction in this case, primarily the strong dependence of the results of mode interaction on the phase angles with which they are incident on the irregular section of a waveguide.

[1] K. V. Avilov, "Refined wide-angle parabolic approximation," in: Waves and Diffraction 85: Ninth All-Union Symp. on Wave Diffraction and Propagation [in Russian], Vol. 2, Izd. TGU, Tbilisi (1985).

[2] A. V. Vagin and N. E. Mal'tsev, Vopr. Sudostr. Ser. Akust., No. 9, 61 (1977).

[3] N. N. Komissarova, Sound Propagation in Irregular Waveguides [in Russian], TsNII RUMB, Leningrad (1982).

[4] F. B. Jensen and W. A. Kuperman, in: Underwater Acoustics and Signal Processing: Proc. of the NATO Advanced Study Institute, Copenhagen, 1980, L. Bjørnø (ed.), D. Reidel, Dordrecht-Boston (1981).

[5] F. B. Jensen and C. T. Tindle, J. Acoust. Soc. Am. 82, 211 (1987).

[6] S. V. Burenkov, Akust. Zh. 35, 19 (1989) [Sov. Phys. Acoust. 35, 11 (1989)].

[7] A. V. Popov, "Calculation of the field in a wide irregular waveguide," in: Propagation of Decameter Radio Waves [in Russian], Nauka, Moscow (1976).

[8] L. M. Brekhovskikh and Yu. P. Lysanov, Fundamentals of Ocean Acoustics, Springer-Verlag, Berlin–New York (1982).

[9] M. J. Buckingham, J. Acoust. Soc. Am. 82, 198 (1987).

Translated by J. S. Wood